

The Dressed State Picture in Quantum Coherence and Interference*

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We investigate the advantageous description of driven multi-level systems in the dressed state representation. A demonstration is given for the cases of subnatural line narrowing and lasing without inversion.

1. Introduction

Rather recently it has been pointed out that externally induced quantum coherence influences basic properties of atomic systems like the natural linewidth for spontaneous emission [1, 2] and the threshold condition for light amplification [3–8]. It here is intended to give an impression how useful the dressed state representation can be for these kinds of problems. This goes back to the fact that dressed states are the eigenstates of the whole Hamiltonian which also includes the interaction of the atoms with the driving field. The dressed states involve the externally generated coherence and are thus more natural than the bare states.

In the simplest case of a two-level system Mollow [9] already pointed out the drastic changes of the spontaneous emission and the absorption spectrum in the presence of an applied driving field.

In the resonance fluorescence spectrum two sidebands appear and we can perfectly understand these in the dressed state picture due to the dynamical Stark splitting. Mollow also predicted coherent amplification in a driven two-level system. This is interesting because of the obvious absence of inversion, though we should call this parametric amplification rather than lasing without inversion.

2. Subnatural Linewidth in a Driven V-Model

There has been a recent investigation of the steady-state spontaneous emission spectrum of the three-level system (Fig. 1) driven by two coherent fields [1, 2]. It turns out that the fluorescence spectrum depends on all decay rates of all levels and not only the one being monitored. The resonance fluorescence spectrum moreover consists of five peaks that can be understood looking at the transitions from the three equally spaced dressed states with some quantum number n for the field to those three with $n - 1$. The effect of the two driving fields on the linewidth of, for example, the center line can be seen in Fig. 2, which features the possibility of a subnatural linewidth. An analytic expression of this linewidth can only be performed in the dressed state picture and only under the assumption of at least one strong Rabi frequency of the two applied coherent fields. We then obtain for the center linewidth Δ for spontaneous emission on the $3 \rightarrow 1$ transition

$$\Delta = W_{31} \cos^2 \theta + W_{21} \sin^2 \theta,$$

where $\tan \theta = g_2/g_1$ and W_{31} , W_{21} and g_1 , g_2 are the spontaneous emission rates and Rabi frequencies of the applied fields on the $3 \rightarrow 1$ and $2 \rightarrow 1$ transition, respectively. It thus becomes obvious that line narrowing occurs of $W_{31} > W_{21}$ and the effect is the more drastic the stronger the field is on the $2 \rightarrow 1$ transition compared to that on the $3 \rightarrow 1$ transition.

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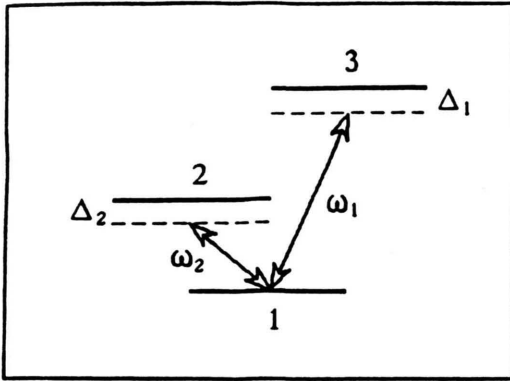


Fig. 1. The V-model. Strong driving on the $2 \rightarrow 1$ transition can lead to line narrowing of the fluorescence spectrum for the $3 \rightarrow 1$ transition.

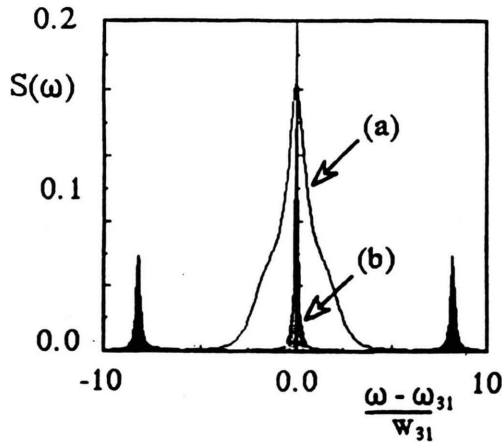


Fig. 2. (a) Power spectrum of the spontaneous radiation emitted by the $3 \rightarrow 1$ transition for the spontaneous emission rate on the $3 \rightarrow 1$ transition being five times as large than that on the $2 \rightarrow 1$ transition, vanishing driving field on the $2 \rightarrow 1$ transition and zero detuning. Apart from small quantitative changes, this spectrum is very similar to the standard Mollow spectrum. (b) Same as curve (a) except four times as strong driving on the $2 \rightarrow 1$ transition as on the $3 \rightarrow 1$ transition. Substantial line narrowing appears.

3. The Origin of Lasing Without Inversion

Figure 3 displays an appropriate scheme to feature lasing without inversion [7, 8]. The applied Raman field with Rabi frequency g_R here establishes a high enough coherence between the two lower and closely spaced levels, that already weak pumping to the upper

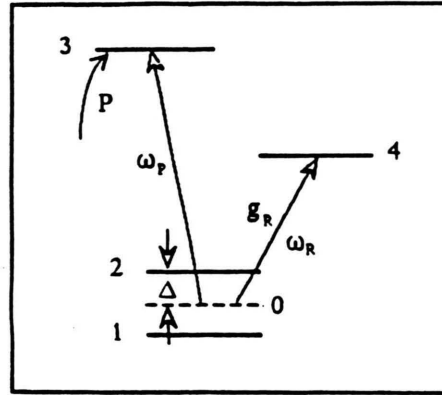


Fig. 3. Schematic energy level diagram of a four-level atom suitable for lasing without inversion; ω_R is the carrier frequency of the applied Raman field (whose Rabi frequency is g_R) and ω_p is the frequency of the tunable probe. 0 is the origin of the vertical energy scale; Δ is half of the frequency ω_{21} , and P is the rate of incoherent pumping to level 3.

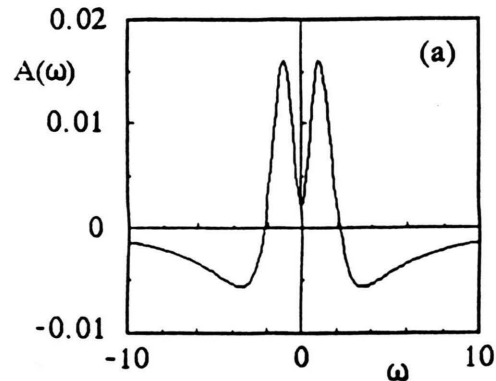


Fig. 4. Absorption spectrum $A(\omega)$ of a weak probe with frequency ω_p plotted as a function of the scaled variable frequency $\omega = (\omega_p - \omega_3)/W_3$, where ω_3 and W_3 are frequency and spontaneous emission rate of level 3.

laser level 3 is enough to give rise to lasing without inversion.

In Fig. 4 the net absorption $A(\omega)$ is given for a probe interacting with the $3 \rightarrow 2$ and $3 \rightarrow 1$ transitions. The occurrence of ranges in ω with positive $A(\omega)$ means amplification. Separating the contributions of $A(\omega)$ in Fig. 4 due to zeroth order populations and zeroth order coherences in the probe field shows that the absorption contribution due to the noninverted populations is competing with the gain contribution due to the induced coherences. In dependence on the

frequency ω one or the other terms is overwhelming, which leads to either gain or absorption.

Looking at the same problems in the dressed state representation yields the surprising fact that the dressed state coherences here give an absorption contribution to the net absorption $A(\omega)$, which approaches zero in the $g_R \rightarrow \infty$ limit [8]. Thus, inversion is necessary between the upper laser level 3 and those dressed states which are coupled to level 3. One of the three dressed states, however, is antisymmetrical in levels 1 and 2 and thus nonabsorptive. A simple dressed state analysis shows that a strong Raman Rabi frequency g_R means a high population in the nonabsorptive dressed state. Thus only little pumping is necessary to establish inversion between level 3 and the two coupled, absorptive dressed states. Since the population in the nonabsorptive state has to be taken into account, dressed state inversion does not auto-

matically mean inversion of the bare states. In short, in the here discussed scheme, lasing without inversion of the bare states may arise from inversion between dressed states [7, 8]*.

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* *Note added in proof:* After this manuscript was prepared, the scheme in Fig. 3 was shown to give rise to lasing without inversion via two mechanisms, inversion of dressed states or solely via the contribution of dressed coherences [10]. Sub-natural line narrowing in the scheme in Fig. 1 was also derived from the relaxation of dressed coherences [11].

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